§1.5. Three types of superconductors

The behavior of superconductors in a magnetic field allows to divide them into three main types. It should be noted that distinctions in behavior in a magnetic field testify to essential distinctions in physics of the microprocesses happening in samples.

In type I superconductors the Meissner state when the magnetic field is pushed out from the volume of a superconductor and is other than zero only in a thin near-surface layer, takes place up to some critical field H_C. If the external field exceeds this value, the sample passes into a normal state.

The effect of pushing out of a magnetic field from a sample can be presented as follows. The shielding currents completely compensating an external magnetic field in a sample give it the magnetic moment. Formally we can speak about the magnetization $\vec{M}$ equal to the magnetic moment of unit volume of a sample. The vector of magnetic induction in a sample is defined by expression $\vec{B} = \mu_0(\vec{H}_e + \vec{M})$. Often the behavior of a superconductor is characterized by dependence of magnetization M on an external magnetic field. Such curve for a type I superconductor is shown in fig. 1.7.

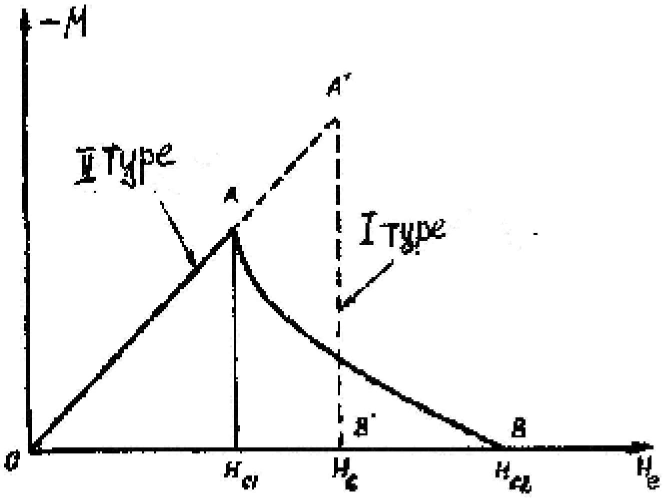

Fig. 1.7. Curves of magnetization of the of the I and II type superconductors having a form of the long cylinder in a longitudinal field

In the same figure the continuous line represents a curve of magnetization of a type II superconductor. In this case the value H_C corresponds to equality of free energies of normal and superconducting states, i.e. this is the value of an external field at which it would be energetically favorable to sample to pass into a normal state. Comparing curves, we can see that in type II superconductors there exists some critical value H_{C1} (smaller than H_C) such that if the external magnetic field exceeds this value it starts getting into the sample. Further with growth of a magnetic field the magnetic moment of a sample monotonously decreases, i.e. the field gets into a sample more strongly. When the external field H_e achieves the value H_{C2}, bigger than H_C, the magnetic moment becomes equal to zero, i.e. the external magnetic field completely suppressed superconductivity. Thus, it is possible to draw a conclusion that the behavior of type II superconductors isn't guided by simple energy reasons as type I superconductors do.

In type III superconductors, or as they are called differently, rigid type II superconductors, the curve of magnetization has absolutely other appearance (a curve 2 in fig. 1.8). Hysteresis character of a curve is obviously visible. After the removal of the external field the magnetic flux remains frozen in the sample.

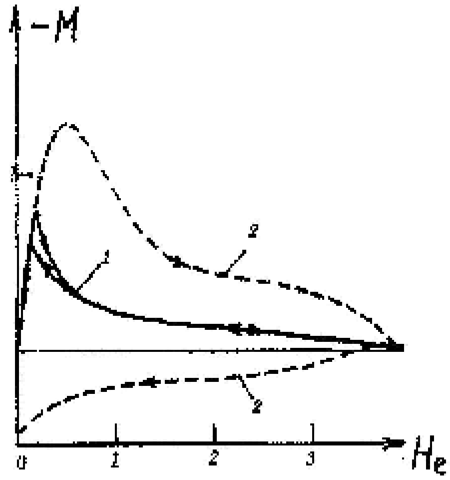

Fig. 1.8. Curves of magnetization of an alloy $Nb_{0,55}Ta_{0,45}$
1- a well annealed sample, 2 - a sample with a big amount of structural defects.

Crystal lattices of type III superconductors contain a large amount of defects which obstruct the moving of vortex threads (we will in detail talk about them later). At careful annealing these defects can be eliminated and the curve of magnetization becomes almost reversible and corresponds to a type II superconductor (a curve 1 in fig. 1.8).

§1.6. Energy gap.

Various experiments, such as tunnel effect, absorption of light and ultrasound, etc., testify that upon transition of substance to a superconducting state a gap arises in its energy spectrum and its value is connected with critical temperature by an approximate formula

$$E_g = 2\Delta \approx 3{,}5kT_c, \qquad (1.3)$$

where Δ is a so-called half-width of a gap.

We will discuss this situation in more detail.

At first let's consider a normal metal. In the main state at $T = 0$ electrons fill all states in Fermi's sphere. In order to get to an excited state, it is enough to move one electron from an originally occupied state ($k \le k_F$) into an empty one ($k' > k_F$). Thus two quasiparticles are formed – an electron with an impulse $k' > k_F$ and a hole in that place where it was earlier. It is natural to measure energy of excitations (quasiparticles) from Fermi's energy:

$$\xi_{k'} = \frac{\hbar^2 (k'^2 - k_F^2)}{2m} \approx \frac{\hbar^2}{m} k_F (k' - k_F) \quad \text{при} \quad k' > k_F \qquad (1.4)$$

$$\xi_k = \frac{\hbar^2 (k_F^2 - k^2)}{2m} \approx \frac{\hbar^2}{m} k_F (k_F - k) \quad \text{при} \quad k < k_F \qquad (1.5)$$

If both impulses lie close to the Fermi's surface, the energy $E_{kk'} = \xi_k + \xi_{k'}$ necessary for their creation is small. In other words, in a metal the excitement can exist with as much as small energy. The dependence $\xi_k(k)$ described by (1.4) and (1.5) is shown in fig. 1.9 by straight lines.

In a superconductor the situation is different. Formulas (1.4) and (1.5) are already unsuitable. The energy necessary for creation of a couple of excitations has to exceed some value called "the energy gap", and the energy of each of the arisen two excitations is described by a formula

$$\varepsilon_k = (\xi_k^2 + \Delta^2)^{1/2} \tag{1.6}$$

and can't be less than a half-width of a gap Δ. The dependence $\varepsilon_k(k)$ described by (1.6) is shown in fig. 1.9.

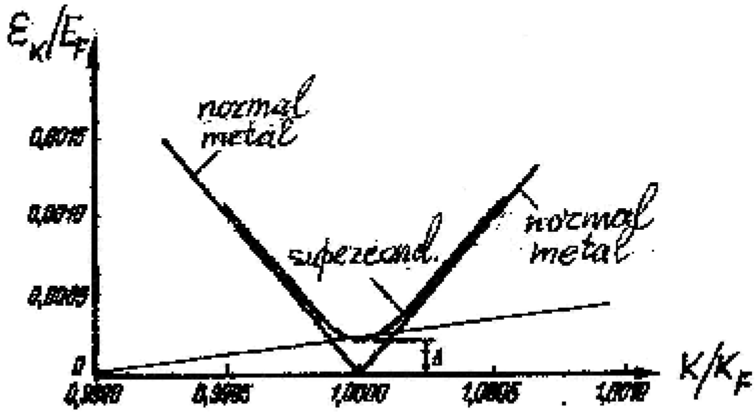

Fig. 1.9. Energy of excitement in normal and superconducting states as function of a wave vector.

Let us consider a crystal lattice with a mass M. Superconducting current can be considered as the collective movement of electronic gas in a lattice. It is possible to tell that the lattice moves with a velocity $\vec{v}$ concerning the electronic gas. "Friction" will reduce this velocity, only if in this gas some excitements arise and the kinetic energy of the lattice turns into their energy. Let there was one excitement with an energy E_k and an impulse $\hbar\vec{k}$. From conservation laws of energy and an impulse we have

$$\frac{1}{2}Mv^2 = \frac{1}{2}Mv'^2 + E_k \;\;,\;\; M\vec{v} = M\vec{v}' + \hbar\vec{k} \tag{1.7}$$

From these two formulae we receive

$$0 = -\hbar\vec{k}\cdot\vec{v} + \frac{\hbar^2 k^2}{2M} + E_k \tag{1.8}$$

Believing the mass of a crystal M infinitely big, we will come to a conclusion that there is the minimum critical value of velocity at which the condition (1.8) can be satisfied

$$v_c = \min\left(\frac{E_k}{\hbar k}\right) \tag{1.9}$$

If a gap exists in a spectrum of excitations then $E_k > 0$, and therefore $v_C > 0$. The inclined straight line in fig. 1.10 has the slope $\hbar k_F v_C / \varepsilon_F$. Thus, in a superconductor the currents with velocities less than v_C proceed without energy loss, i.e. without attenuation. Knowing v_C, we can calculate the critical current density. It can be considerable.

§1.7. One-particle tunneling

Research of tunnel effect gives important information on an energy spectrum of carriers of current. The features of this spectrum in superconductors noted in the previous paragraphs couldn't but affect the tunnel characteristics. Analyzing results of tunnel experiments, Norwegian physicist I. Giaever in 1961 for the first time proved existence of a gap in an energy spectrum of superconductors, for what in 1973 he was awarded by Nobel Prize on physics.

The technique is based on supervision of tunnel current through a thin nonconducting layer dividing two samples. The quantity of the electrons passing through a barrier depends on number of the electrons falling on a barrier, probabilities of tunneling and number of free states on another party of a barrier. We will exclude probability of tunneling from the analysis because it depends on barrier parameters, but not on characteristics of the samples. Then the value of tunnel current will be defined by the density of occupied states on one party from a barrier and the density of free states on another.

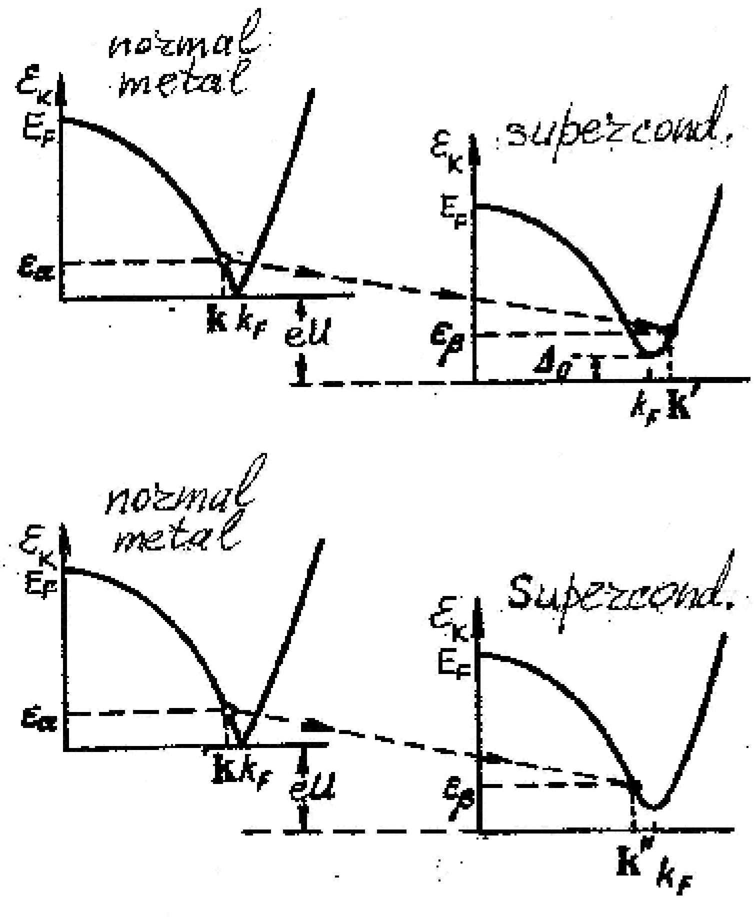

Fig. 1.10. One-particle tunneling between a normal metal and a superconductor.

In fig. 1.10 we can see the one-particle tunneling between a normal metal and a superconductor when the voltage U is applied between them. We note that far from k_F a curve is described by a formula $\varepsilon_k = \left|E_F - \hbar^2 k^2 / 2m\right|$. In the top drawing an electron which was in a state $\vec{k}\uparrow$ under Fermi's surface in a normal metal tunnels through an oxide film into a state $\vec{k}'\uparrow$ over a Fermi's surface in a superconductor. Thus at the left there is a hole corresponding to energy of excitement ε_α. Besides, placing a particle in a state $\vec{k}'\uparrow$, we receive in a superconductor an excitement with energy $\varepsilon_\beta = \sqrt{\xi_{k'}^2 + \Delta^2}$.

Another process is shown in the lower drawing, it coincides with considered one except that the quasiparticle is located in a state $\vec{k}''\downarrow$ under Fermi's level.

Both of these processes can go if energy is conserved: $\varepsilon_\alpha + \varepsilon_\beta = eU$, i.e. at $U > \Delta / e$. At the contact of two normal metals the current exists at any, as much as small, voltage.

V-A characteristics for metal-metal and metal-superconductor are shown in fig. 1.12 (curves 1 and 2).

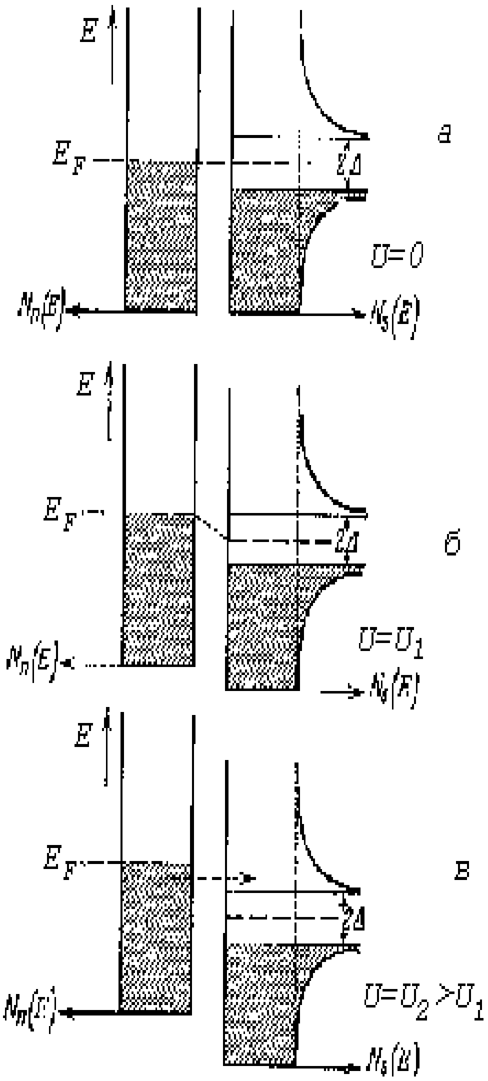

Fig. 1.11. Tunneling between normal metal and a superconductor at T=0 (semiconductor model).

By consideration of the phenomena of tunneling "the semiconductor model" of excitation spectrum is often useful (fig. 1.11). However it is necessary to use it with care since the states "over the energy gap" in this model are actually linear combinations of the quasiparticle states over and under the Fermi's surface. As we will see later the states of single electrons exist as well inside the gap. In the case of semiconductors one-particle states are absent inside the gap.

In this model it is possible to comment the events as follows. In fig. 1.11a we can see a contact at zero voltage. The occupied states are shown by shading. The density of states is plotted on the horizontal axis. The equilibrium state is established at identical Fermi levels in both parts. The transition of electrons from one part into another is absent. The general current is equal to zero. Up to voltage $U = \Delta/e$ the tunnel current is absent as electrons of normal metal can't find suitable states in a superconductor. At $U = \Delta/e$ the current begins abrupt increasing with a vertical tangent. This sharp rise is caused by the high density of states in a superconductor. With a further growth of voltage the curve comes nearer to the tunnel characteristic of two normal metals. At nonzero temperatures in metal there is a quantity of electrons with energy higher than Fermi level, plus the gap in a superconductor decreases. In this case the characteristic assumes an air of the curve 3 fig. 1.12.

The V-A characteristic for contact of two superconductors is schematically shown in fig. 1.13.

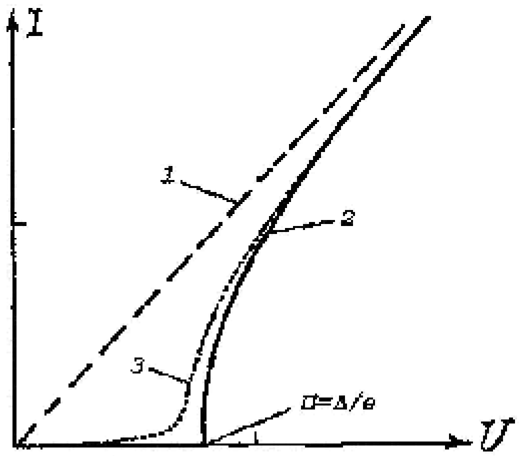

Fig. 1.12. Volt-Ampere characteristics of tunnel contacts.
1 - normal metal / normal metal; 2 - normal metal / superconductor, T=0;
3 - normal metal / superconductor, 0<T<T_c.

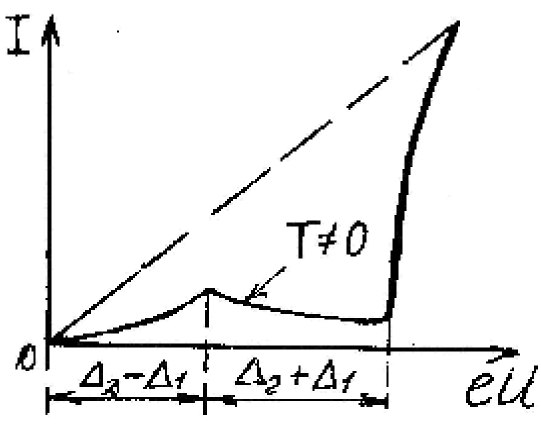

Fig. 1.13. Tunneling between two superconductors.