2.5.3.2. Little and Parks Effect

Let us consider a superconducting film deposited on an insulating substrate in the form of a cylinder of radius R (Figure 2.7). The thickness of the film $d << R$. The uniform magnetic field is applied along the axis of the cylinder.

We find the superconducting transition temperature as a function of applied magnetic field. As before, we assume that $d << \xi(T)$ and $d << \lambda(T)$, and the amplitude $|\psi|$ within the film is constant. As earlier, we write down $\psi = |\psi| \exp(i\varphi(\vec{r}))$, where the amplitude $|\psi|$ does not depend on $\vec{r}$. The density of the Gibbs potential is given by (2.91), and the velocity – by the formula $\vec{v} = \frac{1}{m'}(\hbar\vec{\nabla}\varphi - e'\vec{A})$.

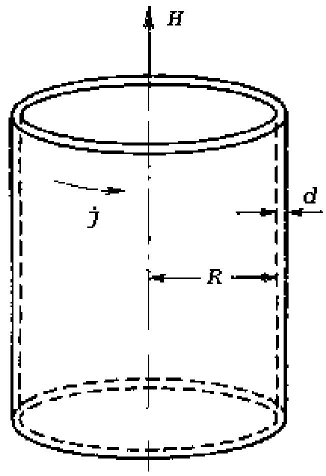

Fig.2.7. The experiment of Little and Parks.

Let us find the dependence of the velocity $\vec{v}$ on the field B. Consider the circulation $\oint \vec{v} \cdot d\vec{l}$, where the integration is carried out along the circumference of the cylinder (radius R). Integrating the expression (2.90), we obtain

$$\oint \vec{v} \cdot d\vec{l} = 2\pi R v = \frac{\hbar}{m'}[\varphi] - \frac{e'}{m'}\oint \vec{A} d\vec{l} \tag{2.94}$$

where $[\varphi]$ - the phase change in the complete circuit around a cylinder. It follows from unambiguity of the function ψ that $[\varphi] = 2\pi n$, where n – any integer. The second term in (2.94) is proportional to the integral

$$\oint \vec{A} d\vec{l} = \int rot\vec{A} \cdot d\vec{\sigma} = \int \vec{B} \cdot d\vec{\sigma} = \Phi = \pi R^2 B \tag{2.95}$$

i.e. to the magnetic flux inside the cylinder. Consequently,

$$v = \frac{\hbar}{m' R}(n - \frac{\Phi}{\Phi_0}), \tag{2.96}$$

where $\Phi_0 = \frac{2\pi\hbar}{e'} = 2 \cdot 10^{-15}$ Wb – the magnetic flux quantum.

At the fixed magnetic field B the flux Φ is also fixed, and the velocity, in agree with (2.96), can have an infinite set of discrete values. However from (2.91) it follows that Gibbs's potential has a minimum only at such value n at which the module of the velocity is minimum. Thus,

$$v = \min\left(\frac{\hbar}{m'R}\left|n - \frac{\Phi}{\Phi_0}\right|\right) \qquad (2.97)$$

i.e. v is periodic function of a field with the period $\frac{\Phi_0}{\pi R^2}$ (for example, to R=0.8 micron there corresponds the period of 10^{-3} T). Knowing the velocity, we will find the value $|\psi|$ minimizing Gibbs's potential (see (2.92)):

$$|\psi|^2 = \beta^{-1}(-\alpha - 0{,}5m'v^2) \qquad (2.98)$$

The solution exists only at $-\alpha > 0{,}5m'v^2$. The temperature of transition T_H decides by a condition $-\alpha = 0{,}5m'v^2$, i.e. T_H is a periodic function of a field B with the period $\frac{\Phi_0}{\pi R^2}$. As α is proportional to ($T - T_C$), it is possible to tell that the curve of dependence $T_H(H)$ consists of a number of parabolic arches. The greatest shift of temperature of transition takes place at $v = \hbar/2m'R$

$$(T_c - T_H)_{\max} = 0{,}55T_C\left(\frac{\xi_0}{2R}\right)^2 \qquad (2.99)$$

From the physical point of view Little and Parks effect can be explained as follows. If a magnetic flux of an external field through an opening of a superconductor isn't equal to an integer of quanta of magnetic flux, in accordance with the law of quantization of a magnetic flux, there has to be a superconducting current in a film, by its field bringing value of a full magnetic flux to an integer of quanta Φ_0. Emergence of this current leads to increase in internal energy in connection with kinetic energy of moving Cooper pairs and energy of the magnetic field created by current. Therefore the transition to a normal state will happen at lower temperature: the stronger are the currents, i.e. the greater is the difference between an external field flux and an integer of quanta of Φ_0, the lower is the transition temperature.

2.5.4. Variation of amplitude of the order parameter in space

2.5.4.1. Formation of germs of superconductivity in a sample

Let us place a superconductor into a strong magnetic field so that the superconductivity is destroyed and a field in a sample is uniform. We will smoothly reduce the field. When the field achieves some value H_{C2}, superconducting areas will start being formed in a sample. We will show that the field H_{C2} is not equal to the critical field H_C, it can be both more or less.

In the area of formation of germs the amplitude of the order parameter $|\psi|$ is small that allows to linearize Ginzburg-Landau equation (2.78)

$$\frac{1}{2m'}\left(-i\hbar\vec{\nabla} - e'\vec{A}\right)^2\psi = -\alpha\psi \qquad (2.100)$$

Besides we will consider that $rot\vec{A} = \mu_0 \vec{H}_e$, where $\vec{H}_e$ is a uniform external field. It is admissible because superconducting currents are proportional to $|\psi|^2$ and in linear approach the amendments to a field caused by them are negligible. The same fact allows to exclude, at a minimization of Gibbs potential, the integral on area out of a sample (see 2.5.1) that confirms the acceptability of the Ginzburg-Landau equations in this case.

The equation (2.100) formally coincides with Schrödinger equation for a particle with a charge e' and mass m' in a uniform magnetic field. In a unlimited environment such particle moves along a field with a constant speed and rotates on a circle in the xy plane with a frequency

$$\omega_c = \frac{e'B}{m'} \qquad (2.101)$$

The energy levels corresponding to own functions of the equation (2.100) have a form

$$-\alpha = \frac{1}{2} m' v_z^2 + (n + \frac{1}{2})\hbar\omega_c, \qquad (2.102)$$

where n is a non-negative integer. The greatest value H_e, i.e. ω_C, at the given α corresponds to a case $v_z = 0, n = 0$, so $-\alpha = 0{,}5\hbar\omega_c$. From here we find H_{c2}. It is possible to show that

$$H_{c2} = \kappa\sqrt{2}H_c \ . \qquad (2.103)$$

Let us discuss a formula (2.103).

At $\kappa > 1/\sqrt{2}$, i.e. $H_c < H_{c2}$, germs of superconductivity can be formed in the thickness of a sample at fields $H_c < H < H_{c2}$. In this phase the field can't be pushed out completely from a sample because at $H > H_c$ the full effect of Meyssner is energetically unprofitable. In this range of field a special, the so-called mixed state, is established in a sample. This state is typical for type II superconductors which we will discuss further.

At $\kappa < 1/\sqrt{2}$, i.e. $H_c > H_{c2}$, at reduction of a field at first the value H_c is reached at which a full Meyssner effect takes place, in other words, we have a type I superconductor.
Thus, the division of superconductors into two types can be made depending on the value of the parameter κ: for type I superconductors $\kappa < 1/\sqrt{2}$, at $\kappa > 1/\sqrt{2}$ we have a type II superconductors.

CHAPTER 3. MAGNETIC PROPERTIES OF SUPERCONDUCTORS.

As it was told in chapter 1, depending on the behavior in a magnetic field superconductors can be divided into three main types. The present chapter is devoted to consideration of the microprocesses happening in type I and type II superconductors and defining their distinction.

§3.1. Type I superconductors. Intermediate state.

In type I superconductors the Meyssner state when the magnetic field is pushed out from the volume of a superconductor and is other than zero only in a thin near-surface region, takes place up to some critical field H_C. If the external field exceeds this value, the sample passes into a normal state. Thus, the dependence of magnetic induction in a sample from intensity of an external magnetic field has the form shown in fig. 3.1.

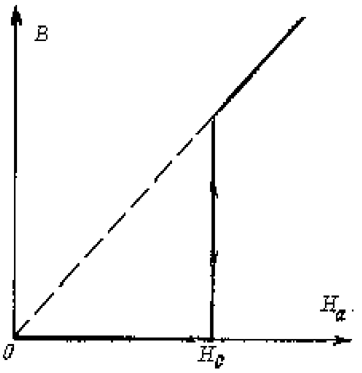

Fig. 3.1. A magnetic field in a sample.

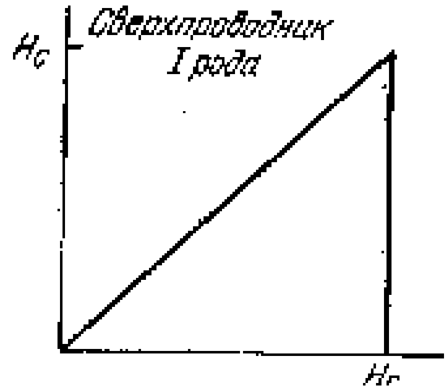

Fig. 3.2. A curve of magnetization of a sample having a core form in a longitudinal field.

The effect of pushing out of a magnetic field from a sample can be presented as follows. The shielding currents completely compensating an external magnetic field in a sample give it the magnetic moment. Though, strictly speaking, the internal areas of a sample don't possess a magnetization, but it is formally possible to speak about the magnetization $\vec{M}$ equal to the magnetic moment per volume unit of a sample. It allows as it was told in §2.2 to enter concept of a vector of intensity of a field of a sample $\vec{H}$, using a relation $\vec{B} = \mu_0(\vec{H} + \vec{M})$. Often as the characteristic of behavior of a sample in a magnetic field the magnetization curve, i.e. the dependence of magnetization M on an external magnetic field, is accepted. Such dependence for a type I superconductor obtained from the schedule of fig. 3.1 is given in fig. 3.2.

As it was shown in §2.2, if the sample has a form of a long core and is placed into the external field $\vec{H}_e$ parallel to its axis, the field $\vec{H}$ is uniform and equal to $\vec{H}_e$ everywhere in the sample. This fact explains the using $\vec{H}_e$ instead of $\vec{H}$ in a formula $\vec{B} = \mu_0(\vec{H} + \vec{M})$ at creation of schedules of fig. 3.1 and 3.2.

In a Meyssner phase magnetic induction $\vec{B}$ in a superconductor is equal to zero, and macroscopic intensity of a magnetic field $\vec{H}$ is equal to an external field, i.e. is other than zero. Then the relation $\vec{B} = \mu\mu_0\vec{H}$ can be executed, only if $\mu = 0$. Thus, the superconductor in a Meyssner phase is ideal diamagnetic with $\mu = 0$.

The threshold, or critical, magnetic field H_C necessary for superconductivity destruction depends on temperature. At a critical temperature T_C the critical field is equal to zero. With decrease of a temperature the value H_C increases that is approximately described by a ratio (see 1.1)

$$H_C(T) = H_C(0)\left(1 - \frac{T^2}{T_C^2}\right) \qquad (3.1)$$

Using expressions $\frac{\hbar^2}{2m'|\alpha|} = \xi^2(T)$ (see the paragraph before 2.46), $\alpha = -\frac{1}{\mu_0}\frac{B_C^2}{n_S(\infty)}$ (see 2.39) and $\lambda^2(T) = \frac{m'}{\mu_0 n_S e'^2}$ (see 2.49), it is easy to come to the expression for H_C

$$H_C = \frac{\hbar}{2\sqrt{2}\mu_0 e\lambda(T)\xi(T)} \quad (3.2)$$

The difference of free energies per volume unit in superconducting and normal states of a sample is equal to $F_N - F_S = \mu_0 H_C^2/2$ (see a formula 1.2)

All told above concerns the samples having a form of a long core, placed in the field, parallel to its axis (fig. 3.3). If to neglect influence of the ends, such geometry provides equality of values of a field on all surface of a sample.

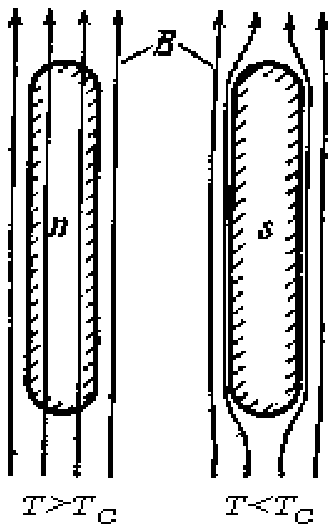

Fig. 3.3. Pushing out of a magnetic field from the sample having a core form when cooling in a magnetic field.