Let us consider a less trivial case, for example, the superconducting sphere of radius a placed in a uniform external magnetic field $\vec{H}_e$ (fig. 3.4). If the field H_e is less than $2H_C/3$, the lines of magnetic induction are forced out from a sample. Distribution of a field outside the sphere is defined by the equations

$$div\vec{h} = 0; \quad rot\vec{h} = 0 \tag{3.3}$$

and the boundary conditions $h \to H_e$ at $r \to \infty$, $h_n|_{r=a} = 0$.

where $h_n|_{r=a}$ - a field component, normal to a surface of the sphere, r - distance from the center of the sphere. The second of the written-down boundary conditions follows from Meissner effect, i.e. from that fact that power lines can't get into the superconducting sphere.

The solution for area outside the sphere has a form

$$\vec{h} = \vec{H}_e + H_e \frac{a^3}{2} \nabla\left(\frac{\cos\theta}{r^2}\right) \tag{3.4}$$

A field component, parallel to a sphere surface, is equal

$$h_\tau|_{r=a} = \frac{3}{2} H_e \sin\theta \tag{3.5}$$

At the poles of the sphere Q and Q' the field is equal to zero, on the equator ($\theta = \pi/2$) the tangential component is maximum and equal to $3/2H_e$. When the external field H_e reaches the value $2/3H_C$, the field on the equator becomes equal to H_C. Therefore, in the range of fields $H_C > H_e > 2/3H_C$ some areas of the sphere pass into a normal state. The other part of the sphere doesn't lose superconductivity (if all sample passed into a normal state, the field in any point would equal $H_e < H_C$, i.e. superconductivity would appear again).

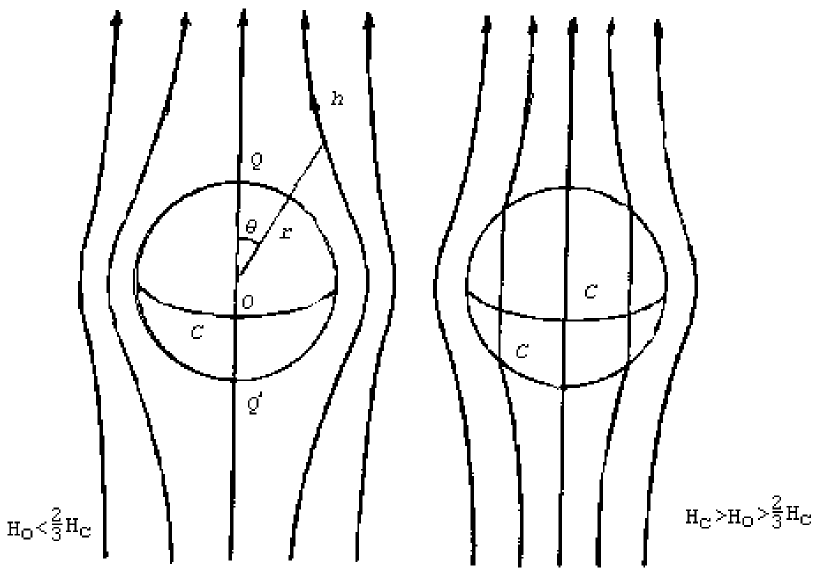

Fig. 3.4. The distribution of a magnetic field near the superconducting sphere.

It would seem that the area near the equator has to pass into a normal state while the central part remains superconducting. Let us show that it can't happen. In this case the field at the border between superconducting and normal phases has to be equal to critical. With increasing the distance from a sphere axis the field decreases and, therefore, the corresponding areas have to remain superconducting that contradicts the initial assumption.

It is possible to show that at all volume inside the sphere will be in a so-called intermediate state, in which normal and superconducting microscopic areas alternate. In the general case of bodies of arbitrary form not necessarily all the volume has to be in an intermediate state. There can remain as well the areas of purely superconducting and normal states adjoining to area of an intermediate state, but only not directly contacting with each other.

We note that the area of fields in which there exists the intermediate state, depends on a geometrical form of a sample. For example, if the sample represents the ellipsoid extended (flattened) along the direction of a field, on the equator the field will differ less (more) from external and the intermediate state will begin at other values of an external field. In particular, in the case of a thin plate in the field, perpendicular its planes, the intermediate state will exist at arbitrary value of the field $H_C > H_e > 0$. If in any field, other than zero, any macroscopic part of the plate remained completely superconducting, as a result of pushing out the field on the edge of this area would be very big that inevitably would lead to transition to a normal state.

Thickness of normal and superconducting layers in an intermediate state are small, therefore at the solution of many tasks it is possible to ignore microscopic structure of layers and to operate only with the relative volume of S-areas ρ, and also the macroscopic quantities B and H. We will calculate values of these quantities in an intermediate state.

The density of free energy is equal

$$F = F_N - \rho \frac{\mu_0 H_C^2}{2} + (1-\rho)\frac{\mu_0 h_N^2}{2} \qquad (3.6)$$

where h_N is a field strength normal areas.

Here the second member represents the energy of condensation in superconducting areas (see a formula 1.2), and the third - magnetic energy in normal areas. We neglected the energy of interfaces of normal and superconducting areas, and also the members considering the distortion of power lines near the film surface (for macroscopic bodies these members are negligible). The magnetic induction by definition is proportional to average field strength in a point and is equal

$$B = \mu_0 \langle h \rangle = \mu_0 (1-\rho) h_N + \mu_0 \rho \cdot 0 = \mu_0 (1-\rho) h_N \qquad (3.7)$$

In variables ρ and $\vec{B}$ the density of free energy is equal

$$F = F_N - \rho \frac{\mu_0 H_C^2}{2} + \frac{B^2}{2(1-\rho)\mu_0} \qquad (3.8)$$

Free energy has a minimum at the fixed values of induction $\vec{B}(\vec{r})$ at each point and the temperature T. We write down the expression for density of thermodynamic potential of Gibbs having a minimum at the fixed temperatures and currents in the coils creating the external field:

$$G = F - BH = F_N - \rho \frac{\mu_0 H_C^2}{2} + \frac{B^2}{2(1-\rho)\mu_0} - BH \qquad (3.9)$$

The full Gibbs potential is equal to the integral over the volume of all space. But as it is told above, the distortion of a field near the surface of the plate is negligible, therefore, at minimization of the Gibbs potential it is possible to be limited to integration (3.9) only over the sample volume.

1) The minimization of Gibbs potential on ρ gives

$$B = \mu_0(1-\rho)H_C \tag{3.10}$$

Comparing (3.10) with (3.7), we obtain $h_N = H_C$.

2) As a result of minimization on B we receive

$$B = \mu_0(1-\rho)H \tag{3.11}$$

This ratio can also be received, having substituted in (3.9) H, found from (2.7), and then having minimized on B.

From comparison (3.11) with (3.10) we receive $H = H_C$, i.e. the value of the field H is constant and equal to H_C in all volume of a sample.

Having written down this fact in a form $H_C^2 = H^2$ and having taken a gradient from both parts, we will use a formula $grad(\vec{a}\cdot\vec{b}) = \vec{a}\times rot\vec{b} + \vec{b}\times rot\vec{a} + (\vec{a}\nabla)\vec{b} + (\vec{b}\nabla)\vec{a}$ and receive $0 = \nabla H^2 = 2(\vec{H}\nabla)\vec{H} + 2\vec{H}\times rot\vec{H}$. Since inside the sample $rot\vec{H} = 0$, then $(\vec{H}\nabla)\vec{H} = 0$, i.e. the vector $\vec{H}$ doesn't change along the line of magnetic induction, therefore, all these lines are straight lines. Generalizing this result, it is possible to tell that for any structure of an intermediate state the borders of phases have to be parallel to a magnetic field (see fig. 3.4).

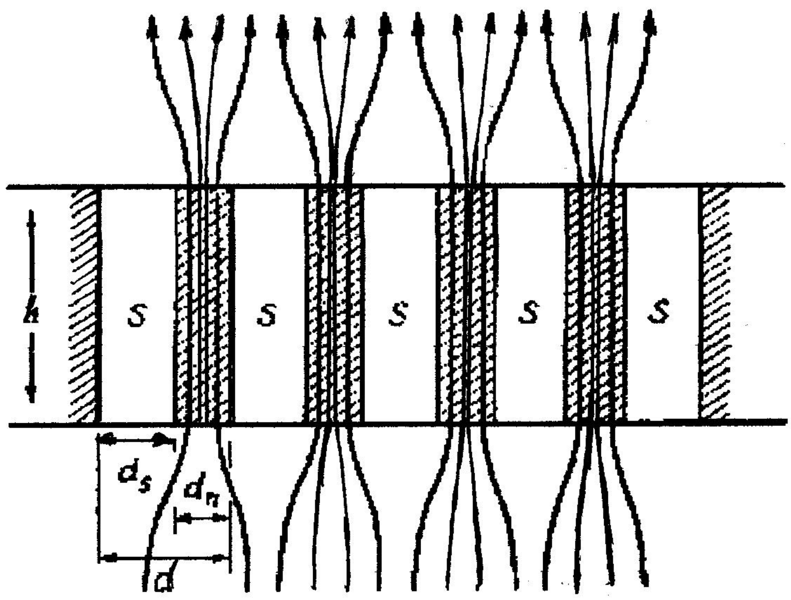

Figure 3.5. The distribution of power lines in the plate in the field perpendicular to its plane.

Let us consider the case of a flat plate in the field perpendicular to its plane. The distribution of normal (N) and superconducting (S) areas are shown in Figure 3.5: area N and S form layers perpendicular to the plane of the drawing. Power lines pass only through N. The magnetic induction at the interfaces should be equal to $\mu_0 H_C$, and inside S-regions - zero.

The part of the volume occupied by S-regions $\rho = d_S/(d_S + d_N)$, for such a simple configuration can be found just from the conservation of flow. Away from the film field is uniform, $h = H_e$, and the magnetic flux is $\mu_0 S H_e$ (S - the surface area of the film). In the film this flow passes through the area of the N-region equal to $S(1-\rho)$, and the field in the N-area as shown above, is uniform and equal to H_C. Consequently, $\mu_0 S H_e = \mu_0 S(1-\rho)H_C$ whence

$$\rho = 1 - \frac{H_e}{H_C} \tag{3.12}$$

The more is the external field, the smaller is the fraction of the volume occupied by the S-regions.

Figure 3.6 shows a picture of an intermediate state of a thin film.

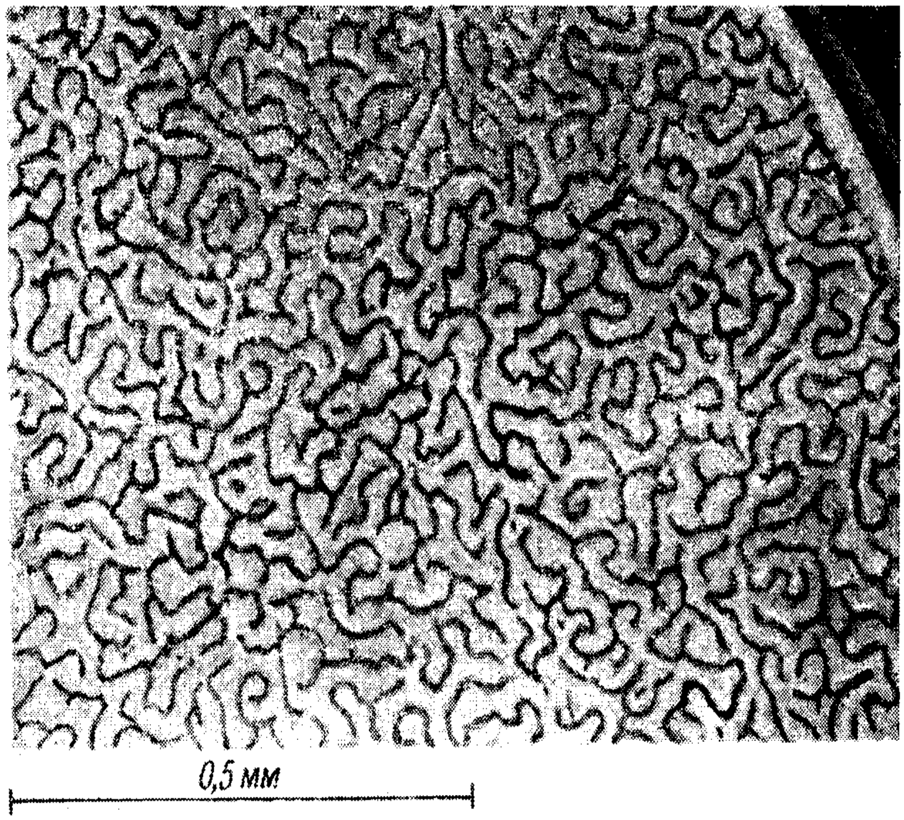

Figure 3.6. Photo of the structure of the intermediate state of a thin film.
The thickness of the film is 7 microns. Dark areas - superconducting regions.

§3.2. The energy of the boundary between the phases.

In the chapter II it has been shown that the external magnetic field required for the destruction of superconductivity in the superconducting film having a thickness smaller than the penetration depth, is greater than the corresponding values for the bulk superconductor. Therefore, it would seem, it is energetically favorable for the sample to split into thin layers of alternating S and N, and thus it could remain in that state at $H_e > H_C$. However, this does not occur and the sample goes into the normal state. So, splitting into thin layers is energetically unfavorable. The reason for this is that the formation of the interface is associated with an additional energy, which is positive for type I superconductors. Later we will see that this energy can be negative (in type II superconductors).

Let us consider a surface energy in detail. Figure 3.7 schematically shows the boundary between the normal and superconducting phases.

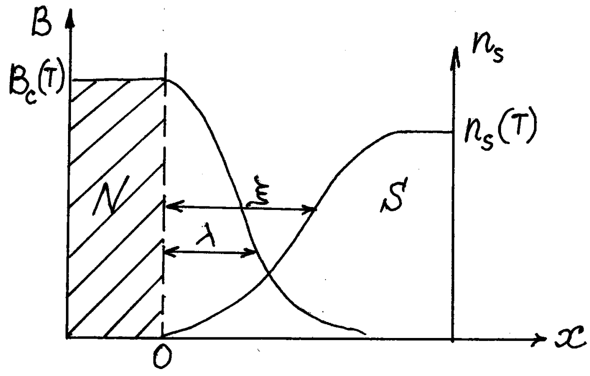

Figure 3.7. The distribution of the magnetic field and the density of Cooper pairs near the boundary of the normal and superconducting phases.

In the normal region (x <0), the magnetic field is greater than or equal to the critical value, and in the superconductor decreases to zero at the London depth λ. In the interior of the superconductor ($x \to \infty$), the density of Cooper pairs is equal to its equilibrium value. Ginzburg-Landau theory (§2.5) shows that the density of Cooper pairs can not change abruptly, the characteristic length of change is the coherence length ξ. Therefore, in a superconductor the density of Cooper pairs and the magnetic field changes as shown in Figure 3.7. Let us assume that in the given superconductor

$$\xi > \lambda . \qquad (3.13)$$

The energy of the interface is determined by the distinction of the picture near the interface from the situation where immediately to the right of the boundary the field is zero, and the density of superconducting pairs is equal to the equilibrium one. Let us find the energy E_B associated with the expulsion of the magnetic field and the energy E_C released by the condensation of Cooper pairs.

In normal region $E_B = E_C = 0$, and in the depth of the superconductor $E_B = E_C = \mu_0 H_C^2 V/2$ (see Eq. (1.2)), where V - the volume. In the border area, both energies do not reach their full values. The positive energy of pushing is smaller than it would be in the case of full pushing by the value of $\Delta E_B = S\lambda\mu_0 H_C^2/2$, where S is the area of the border. The negative energy of pairs condensation is also reduced (by modulus), as in the boundary layer the density is less than the equilibrium value. The reduction of the condensation energy is equal to $\Delta E_C = S\xi\mu_0 H_C^2/2$.

We won ΔE_B in the amount of energy and lost ΔE_C. Therefore, the additional energy required for the formation of the border is

$$\Delta E_C - \Delta E_B = (\xi - \lambda) S\mu_0 H_C^2/2 \qquad (3.14)$$

and in the case of $\xi > \lambda$ is positive. Therefore, the formation of boundaries is energetically unfavorable.

From (3.14) it follows that the sign of the interface energy is determined by the ratio between the London length λ and the coherence length ξ.