3.3.3. The magnetization curve of type II superconductors.

Suppose that the sample has a shape of a long core and is placed into an external field $\vec{H}_e$ parallel to its axis. If the field is sufficiently small, threads, if exist, are rarely located, and their interaction can be neglected. Then the Gibbs potential per unit volume (1 m^2 x 1 m) is equal to

$$G = F_S + n_L F - BH \qquad (3.36)$$

where n_L - the number of threads per 1 m^2, F - the energy per unit length of the thread of (3.31).

It has repeatedly been said that at the considered geometry the magnetic field $\vec{H}$ at all points, both outside and inside the sample, is equal to the external field $\vec{H}_e$. Therefore, instead of $\vec{H}$ in the formulas we will write $\vec{H}_e$.

Since each filament has a magnetic flux Φ_0, the induction equal to the magnetic flux per unit area is given by

$$B = n_L \Phi_0 \qquad (3.37)$$

that allows us to write the Gibbs potential in the form

$$G = F_S + B\left(\frac{F}{\Phi_0} - H_e\right) \qquad (3.38)$$

If $H_e < \frac{F}{\Phi_0}$ the most favorable energy situation corresponds to $B = 0$; the magnetic field is expelled from the sample (the Meissner effect). If, however, $H_e > \frac{F}{\Phi_0}$ the advantageous situation is $B \neq 0$. Thus, the critical field H_{C1} is

$$H_{C1} = \frac{F}{\Phi_0} = \frac{\Phi_0}{4\pi\lambda^2\mu_0}\ln\frac{\lambda}{\xi} \qquad (3.39)$$

Compare the value of the field H_{C1} with the field $H_C = \frac{\hbar}{2\sqrt{2}\mu_0 e\lambda(T)\xi(T)}$ (see 3.2). The ratio of these fields is equal

$$\frac{H_{C1}}{H_C} = \frac{1}{\sqrt{2}}\frac{\xi}{\lambda}\ln\frac{\lambda}{\xi} \sim \frac{\xi}{\lambda} \qquad (3.40)$$

and may be very small.

The magnitude of the field H_{C2}, in which the small superconducting areas are starting being formed in the sample, as it has been shown in §2.5.4, is govern by the Ginzburg-Landau parameter $\kappa = \lambda(T)/\xi(T)$:

$$H_{c2} = \kappa\sqrt{2}H_c = \frac{\Phi_0}{2\pi\mu_0\xi^2(T)} \qquad (3.41)$$

From (3.41) it is clear that the field H_{C2} corresponds to the situation when the cores of the vortices begin to overlap.

The value H_{C3} is associated with a surface superconductivity and is determined by the creation of superconducting jerms at the surface of the sample. The calculation based on the Ginzburg-Landau equations leads to the following expression

$$H_{c3} = 1{,}7H_{c2} = 2{,}4\kappa H_C \qquad (3.42)$$

The equilibrium density of the threads in the sample we will found from the condition of minimum of Gibbs potential for a large number of threads. Consider the case when the external field slightly exceeds the critical value. Abrikosov showed that the minimum of Gibbs potential corresponds to the periodic structure. A detailed calculation shows that this is a triangular lattice (see Fig.3.12). For a small excess of H_e over H_{C1} the density of the vortex filaments is small, so it is necessary to take into account only the interaction of the nearest neighbors (see 3.35)

$$G = F_S + n_L(F + \frac{Z}{2}U_{12}) - BH = F_S + B\left(H_{C1} - H_e + \frac{1}{2}Z\frac{\Phi_0}{2\pi\lambda^2\mu_0}K_0\left(\frac{d}{\lambda}\right)\right) \qquad (3.43)$$

where Z - the number of nearest neighbors (in a triangular lattice, Z = 6), K_0 - Bessel function (Hankel) of zero order of imaginary argument, d – the distance between adjacent vortices. Given that the area of each triangle $S = d^2\sqrt{3}/4$ contains a flux of $3\cdot\Phi_0/6 = \Phi_0/2$ (in the triangular plane lattice each node is divided by 6 cells – see Fig. 3.12) from the formula $B = \Phi/S$ we can find $d = \sqrt{\frac{2\Phi_0}{B\sqrt{3}}}$.

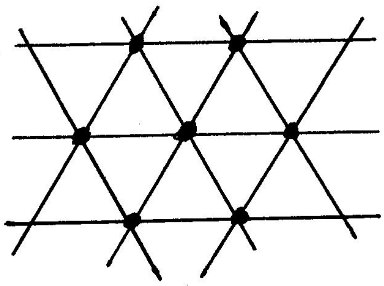

Figure 3.12.

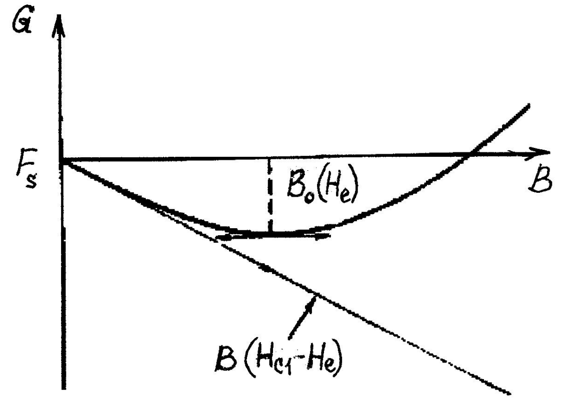

Figure 3.13. The dependence of the thermodynamic potential G on the induction B.

The dependence $G(B)$ is shown in Figure 3.13. For a small excess of H_e over H_{C1} the last term in (3.43), corresponding to the interaction, is small and the slope is negative, with an increase of the induction B the interaction is growing, but rather slowly. This is due to the fact that when $d > \lambda$ the value of $K_0\left(\frac{d}{\lambda}\right)$ takes the form $K_0\left(\frac{d}{\lambda}\right) \approx \exp\left(-\frac{d}{\lambda}\right) = \exp{-1.07\sqrt{\frac{\Phi_0}{B\lambda^2}}}$.

Therefore, for small B the interaction is small. However, at high B the contribution of this term is predominant, what leads to the growth of the function $G(B)$. Therefore, at a certain value

$B_0(H_e)$ the function $G(B)$ reaches a minimum. This value will be the equilibrium value of the induction in the field $\vec{H}_e$.

The theoretical curve $M(H_e)$ for the vortex model is shown in Figure 3.14 (solid curve). At $H_e = H_{C1}$ it has an infinite slope. The dashed curve relates to a laminar model. The crosses show the results of the experiment.

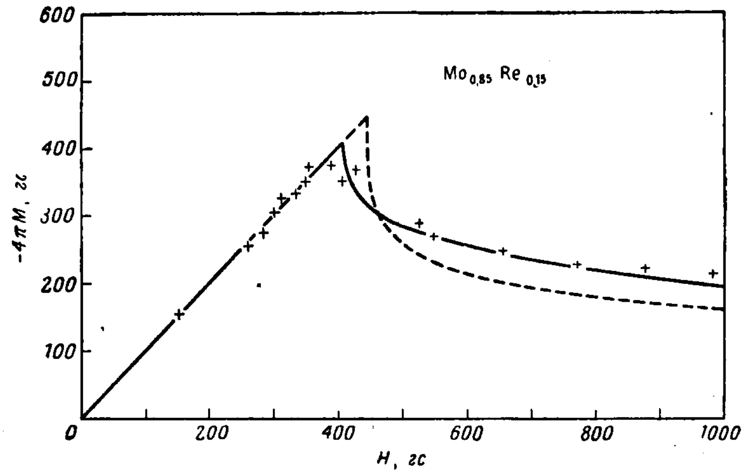

Figure 3.14. The magnetization curves of the type II superconductor having a form of a long cylinder: solid line - vortex model, dashed - laminar model, crosses - the experimental values.

3.3.4. Laminar structure

Let us find Gibbs potential and critical field for the structure shown in Figure 3.15. It is a system of thin equidistant N-layers. We denote by d the period of the structure and assume that the layers are perpendicular to the x-axis. Within the N-layers the superconductivity is suppressed, and in superconducting areas it is characterized by the usual value of the density of superconducting electrons.

Suppose, as before, that $\xi << \lambda$. We place the origin of coordinates in the middle between the neighboring N-layers. Field $h(x)$ is parallel to the z axis and everywhere, except the narrow N-layers, satisfies the equation

$$h = \lambda^2 \frac{d^2 h}{dx^2} \tag{3.44}$$

The solution of equation (3.44) has the form

$$h(x) = H_m \frac{ch(x/\lambda)}{chP} \tag{3.45}$$

where $P = d/2\lambda$, H_m - the field within the N-layers.

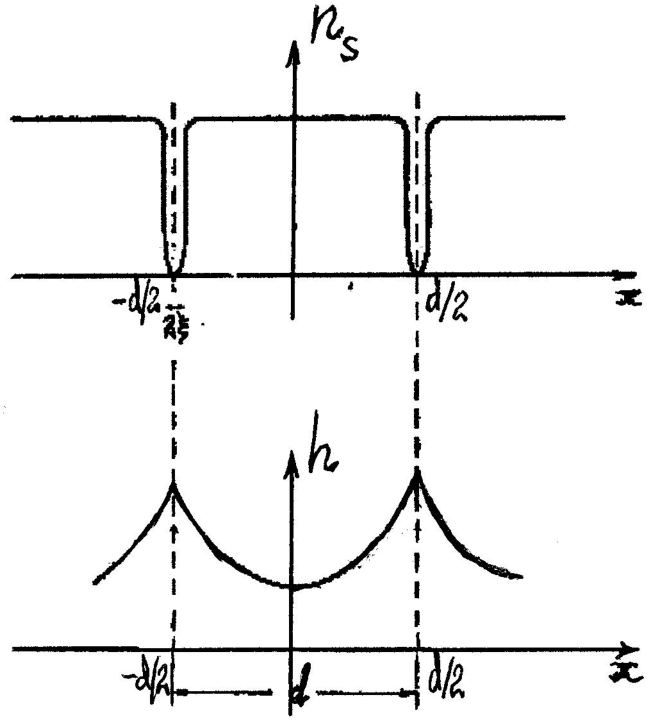

Figure 3.15. The structure of the Shubnikov phase in a laminar model.

Let us find the value of the magnetic induction

$$B = \mu_0 \langle h \rangle = \mu_0 \frac{2}{d} \int_0^{d/2} H_m \frac{ch(x/\lambda)}{chP} dx = \mu_0 H_m \frac{thP}{P} \qquad (3.46)$$

The free energy density in the S-regions, according to (3.18) is equal to

$$F_1 = \frac{2}{d} \frac{\mu_0}{2} \int_0^{d/2} \left[h^2 + \lambda^2 \left(\frac{dh}{dx} \right)^2 \right] dx = \frac{\mu_0 H_m^2}{2} \frac{thP}{P} \qquad (3.47)$$

To this quantity we must add the energy of creating the N-layers (per 1 m^3)

$$F_2 \approx \frac{\mu_0 H_C^2}{2} \frac{2\xi}{d} = \frac{\mu_0 H_C^2}{2} \frac{1}{P\kappa} \qquad (3.48)$$

where $\kappa = \lambda/\xi$ - the Ginzburg-Landau parameter.

To move to the Gibbs potential we need to subtract $BH = \mu_0 H H_m \frac{thP}{P}$:

$$G_{ламин} = \frac{\mu_0}{2} \left(H_m^2 \frac{thP}{P} + \frac{H_C^2}{P\kappa} - 2HH_m \frac{thP}{P} \right) \qquad (3.49)$$

Taking into account the comments to (3.36), we will write in formulas H_e instead H.

Minimizing G by H_m, we find that the minimum is reached at $H_m = H_e$

$$G_{\min} = \frac{\mu_0}{2P} \left(\frac{H_C^2}{\kappa} - H_e^2 thP \right) \qquad (3.50)$$

At $H_e < H_C / \sqrt{\kappa}$ the minimum corresponds to $P = \infty$, i.e. there is a full Meissner effect.

When $H_e > H_C/\sqrt{\kappa}$ the minimum is reached at a finite value of P.

Thus, the critical field in the laminar model is equal to

$$H_{C1_{ламин}} = \frac{H_C}{\sqrt{\kappa}} \qquad (3.51)$$

Comparing this value with the critical field for the threads H_{C1} from (3.40)

$$H_{C1} = \frac{1}{\sqrt{2}}\frac{\ln\kappa}{\kappa}H_C \qquad (3.52)$$

we come to the conclusion that in this case (when $\xi << \lambda$, i.e. $\kappa >> 1$) the critical field for the filaments is less than for the laminar structures. From the analysis it follows that in the range $H_{C1} < H_e < H_C/\sqrt{\kappa}$ the laminar state is a Meissner one, and the energy of the vortex state in this range is less than of Meissner one. In other words, the vortex state is energetically more favorable than the laminar.

We can show that this is true as well in the field range $H_e > H_C/\sqrt{\kappa}$, i.e. throughout the entire range of fields $H_e > H_{C1}$ the vortex state is the most energetically favorable.

CHAPTER 4. CRITICAL CURRENTS IN SUPERCONDUCTORS

In §1.4 it was shown that there exists a critical velocity of Cooper pairs, and hence a critical current density. When the current density is below the critical the system of Cooper pairs can not interact with the lattice. If the current density is higher than the critical value the Cooper pairs are being destroyed and the superconductivity disappears.

The problems related to the critical currents are essential for technical applications of superconductors. While type II superconductors retain superconductivity in high magnetic fields, for their technical applications it is just very important that they could carry without losses the currents of sufficiently high values. As we will see, this problem can be solved in type III superconductors.

§4.1. Critical currents in type I superconductors

The simplest from a geometrical point of view example is a wire of circular cross section, through which the current flows. When the current is weak, the wire should be in the Meissner phase, i.e. the field within the sample is zero. It follows that inside the sample the current can not flow, because it would create a magnetic field inside the superconductor. Therefore, the current flows only in a thin surface layer, into which the magnetic field can penetrate. These currents, in order to be distinguished from screening ones, will be called the transport currents.

Figure 4.1 shows the distribution of the transport current density and magnetic field in the cross section of round wire. With increasing of the current the magnetic field increases. In 1916 F. Silsbi suggested that the critical current density is achieved when at the surface of the sample the magnetic field reaches a critical value. This assumption was brilliantly confirmed by experiment. With hypothesis of Silsbi one can also find critical currents of superconductors in an external magnetic field. To do this, we have to add the external field and the field of the transport current. The critical value of the current corresponds to the moment when at some point of the sample the field is equal to the critical value.

The critical current density can be very high (~10^7 A / cm^2), but due to the small thickness of the surface layer the total current is not high.

Consider the wire with a radius a, through which the current J flows. The field on the surface of the wire is $h(a) = J/2\pi a$. If $h(a) < H_C$ the whole wire can be in a superconducting state. This condition defines a critical current value $J_C = 2\pi a H_C$. If $J > J_C$ we have $h(a) > H_C$ and the wire near the surface must go to a normal state. However, the whole wire can not be in a normal state, because in this case the current would distributed uniformly over the cross section of the wire and the field near the axis would be less than critical. To verify this, we find the field inside the wire at a distance r from the axis when the current is flowing with a current density j in all points of the cross section. Using Stokes theorem on the circulation of the magnetic strength we obtain:

$$h(r) = \frac{J_r}{2\pi r} = \frac{j\pi r^2}{2\pi r} = \frac{jr}{2} \qquad (4.1)$$

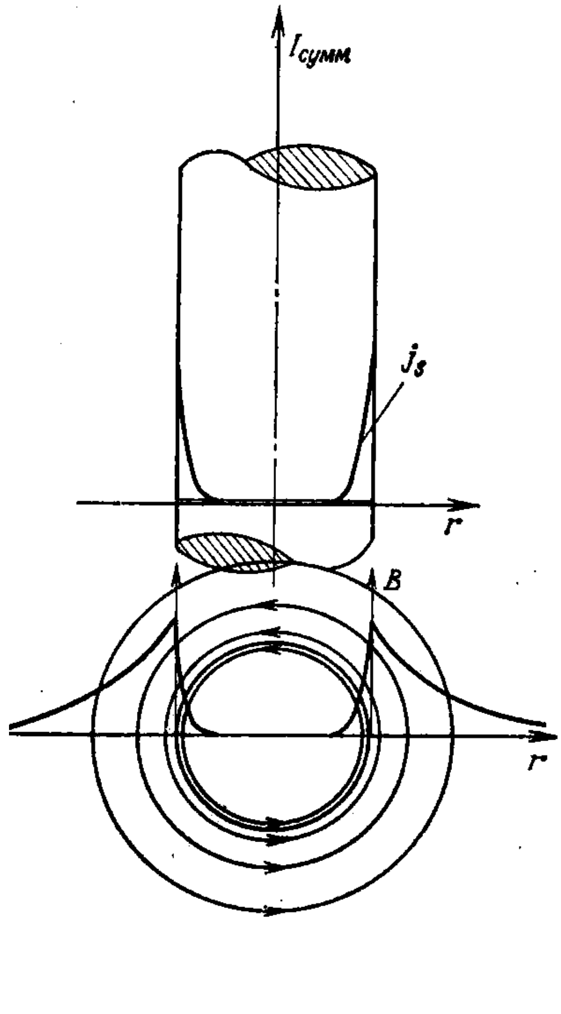

Figure 4.1. Distribution of current density and magnetic field in the superconducting wire with a transport current.

From (4.1) it follows that near the axis ($r \approx 0$) the field is weak. Therefore, the transition of these areas to the normal state is energetically unfavorable. Hence, the outer region of the wire ($R < r < a$) is normal and the inner part ($0 < r < R$) should be either in a superconducting, or in an intermediate state. The radius of interface R corresponds to the condition $h(R) = H_C$. Thus, the current flowing in the inner part is

$$J_1 = 2\pi R H_C = \frac{R}{a} J_C < J_C \tag{4.2}$$

The rest current $J - J_1$ flows in the outer part of the section. Since it is normal, then the current flow through it requires a voltage along it. Consequently, the inner part can not be completely superconducting since it would short-circuit the poles of the generator. Thus, the inner part of the wire is in the intermediate state. But the variant of alternating flat layers as in Figure 3.5, does not solve the problem, since at the interfaces of superconducting and normal areas the magnetic field must equal to critical, and in a flat version it is not fulfilled.

The detailed calculation gives the structure shown in Figure 4.2. Once the current reaches a critical value, the wire jumps to the state in which the superconducting cells reach the surface. With further increase of the current in the wire something like the normal phase shell arises covering the the intermediate state core; with increasing current, the thickness of the shell increases, and the superconducting core region decreases. At all points on the N-S interfaces $h = H_C$, i.e. the closer to the axis the higher is the current density (see. 4.1), what can be obtained by increasing the proportion of the superconducting phase.

From the above, in particular, it follows that the current of $J > J_C$ can not exist in a superconducting ring without a power supply.

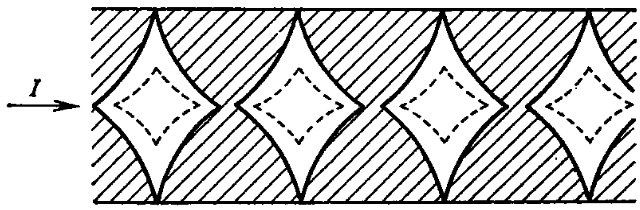

Figure 4.2. The structure of the intermediate state of the round cross-section wire with a transport current. The normal area is shaded.