§4.2. Critical currents in type II superconductors.

In weak magnetic fields and at low transport currents, they behave the same way as type I superconductors, i.e. push the magnetic field and current into a thin surface layer. If the magnetic field on the sample surface is higher than H_{C1}, the sample is in a mixed state, i.e. it is penetrated by the filaments of magnetic flux. It turns out that in this state at any, even very small, transport currents the sample has a finite resistance.

To understand the cause of this phenomenon, let us consider a rectangular plate along the surface of which the electric current flows, and due to the perpendicular external magnetic field the plate itself is in a mixed state (Figure 4.3).

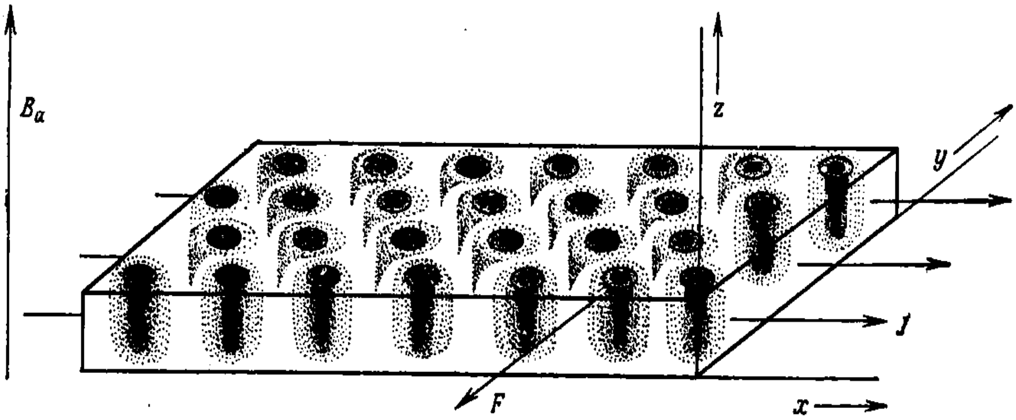

Figure 4.3. The mixed state in the presence of a transport current

An important conclusion from this consideration is the fact that in these conditions the current is distributed evenly over the cross section of the plate and is not limited to a thin layer near the surface. With the penetration of magnetic flux into the sample the transport current can also penetrate into the interior of the superconductor.

This creates an extremely important interaction between the transport current and the threads of magnetic flux. The Ampere force $F = JBl$ directed perpendicular to a field and current acts on a thread. Under the influence of this force threads move perpendicular to their axis what leads to energy losses. This energy is drawn from the energy of the transport current in the sample and thus there is a voltage that corresponds to the appearance of resistance.

The losses are caused by two main mechanisms.

1. When vortex moves the magnetic field at the point changes, an alternating electric field arises which accelerates the unpaired electrons, which later give energy to the lattice.

2. At the movement of threads there is a continuous process of disintegration and formation of Cooper pairs. If the thread moves so slowly that the distribution of pairs remains equilibrium, the energy spent for a rupture of pairs on the forward front of a vortex is released again behind it at formation of pairs with the result that there is no energy loss. But at rather fast movement of a vortex the equilibrium density of pairs doesn't have enough time to be reestablished and the energy is dissipated.

Let us now consider the question of the critical current in the absence of an external magnetic field. Consider again the wire of radius a along which the current J flows. A field on a surface of a wire equals $h(a) = J/2\pi a$. If $h(a) < H_{C1}$, the whole wire can be in a superconducting state.

This condition defines a critical current value $J_C = 2\pi a H_{C1}$. When $J > J_C$ the field at the surface exceeds H_{C1}, and a region at the surface of the wire must go to the Shubnikov phase. Since the magnetic field lines of the transport current are the concentric circles, the vortex filaments are also formed in the form of closed circles. At the beginning they have a radius a, but then for reasons of minimizing the energy, i.e. length, they are compressed to the wire axis, and finally disappear. The formation of vortices, their compression and disappearance occurs continuously, so there is a constant transition of energy into heat. Since $H_{C1} < H_C$ the critical currents in type II superconductors are lower than in similar samples of the type I.

It should be said that in the intermediate state of type I superconductors under the influence of sufficiently strong transport currents the movement of areas can also occur, what leads to the emergence of resistance.

§4.3. Type III superconductors.

In paragraph 4.2 the important result is formulated: if a type II superconductor is in the mixed state (a Shubnikov phase), as much as small transport currents lead to the movement of vortices. In other words, the critical current of a superconductor in a Shubnikov state is equal to zero. Nonzero critical currents can be received, only if to carry out the fixing (pinning) of vortex threads on certain sites of substance obstructing their traffic. Type II superconductors containing such centers of fixing of vortices are called rigid type II superconductors or type III superconductors.

Various impurities, violations of structure, defects can serve as points energetically preferable for vortices. As it often happens, the samples, ideal from the point of view of the theory, aren't the best in the relation of their practical application.

Curves of magnetization of an alloy of Nb and Ta, typical for type III superconductors of are given in fig. 4.4. Very careful annealing of a sample allows to receive very uniform solid solutions which possess almost reversible curve of magnetization, characteristic for type II superconductors (a curve 1). If this alloy is subjected to deformation (for example, by drawing during the process of wire production), the set of defects in a lattice is formed which can be the centers of a pinning of vortices. Thus the curve of magnetization takes absolutely other form (a curve 2).

It is possible to note the following features:

a) significantly increased values of magnetization,

b) total absence of reversibility,

c) after the removal of the external magnetic field the flux remains "frozen" in a sample,

d) the top critical field H_{C2} remains invariable.

These facts can be easily explained qualitatively. To a field H_{C1} we don't observe anything new: the sample is in the Meyssner phase almost not sensitive to presence of violations. Since a field H_{C1} vortices get from a sample surface into its volume. However the pinning doesn't allow them to fill evenly at once all the sample as it would be in a uniform material. Therefore vortices settle down in a near-surface layer. In the area of their placement the shielding currents can flow. Expansion of an area in which vortices are situated, leads to increasing of the shielding current in comparison with a Meyssner phase that leads to increasing of the value of magnetization M.

In other words, the penetration of vortex threads into a near-surface layer increases the effective thickness of the shielding layer and by that the total shielding current.

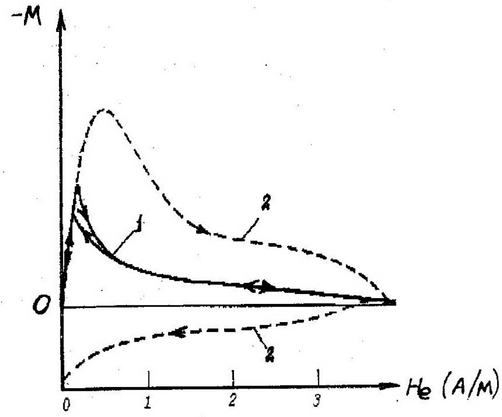

Fig. 4.4. Magnetization curve of an alloy $Nb_{0.55}Ta_{0.45}$*: 1-a well-annealed sample, 2 – a sample with a large amount of defects.*

At decrease of an external field in the absence of a pinning a part of vortices would leave a sample, and the density of distribution of vortices on the section of a sample would decrease to equilibrium value. In the presence of a pinning vortices are more or less strongly fixed on defects of a lattice. Therefore at decreasing field they leave the sample with difficulty, what determines the lack of reversibility.

Even at zero external field the sample contains a number of vortices that provide the "frozen" magnetic flux directed along the external field.

Figure 4.5 shows the entire hysteresis loop for the same alloy.

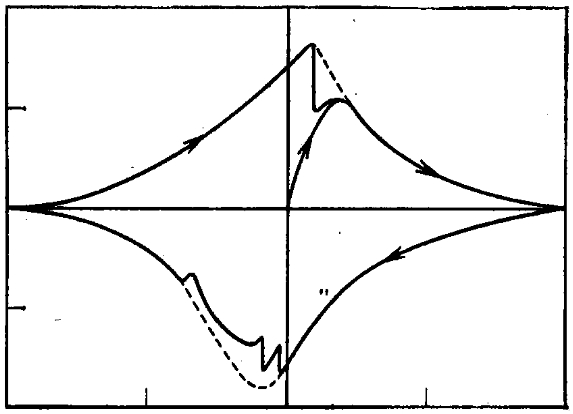

Fig.4.5. The entire cycle of magnetization.

It is clear that there must be some critical pinning separating the two possible modes. For small values of the pinning, where it can be neglected, we have a type II superconductor, in which the vortices fill the entire cross section of the sample. At a high pinning vortices are located near the surface. How and when there is a transition from one mode to another with a gradual change of the pinning parameter?

Calculations show that there exists a critical pinning separating the two possible modes of penetration of an external magnetic field into the medium. If it is exceeded, at any value of the external field there is a “near the surface” current configuration of finite length, fully compensating the external field in the depth of the sample, i.e. the situation is similar to type III superconductor. If pinning is less than critical, such a situation is realized only up to a certain value of the external field. For higher values of the field, it penetrates into the medium at infinite depth, that resembles the situation in type II superconductors.

CHAPTER 5. THE PHASE COHERENCE - JOSEPHSON EFFECTS.

§5.1. Stationary and non-stationary Josephson effects

In the chapter "Basic facts" we talked about the Josephson effects, caused by tunneling of Cooper pairs through an insulating layer. B. Josephson was the first who considered this effect in 1962. For these works in 1973 he was awarded by Nobel Prize. He showed that the tunneling of Cooper pairs becomes essential at a thickness of barrier of 10-20 angstrom. Besides, he predicted some unusual and interesting phenomena taking place when electrons tunnel in pairs. Subsequently all his predictions were excellently confirmed on experiment. Besides their basic value for understanding of superconductivity the Josephson effects (it is accepted to call this complex of the phenomena) give the opportunities for carrying out the most exact measurements. We will emphasize that they play especially important role in the processes occurring in the high temperature ceramic superconductors because in them Josephson contacts already exist naturally (contacts between granules). For this reason these substances sometimes are called Josephson mediums.

The stationary effect of Josephson is a percolation of not fading superconducting current through a thin isolating layer at a zero voltage on contact. The magnitude of this current is determined by the phase values φ_1 and φ_2 on different sides of the contact and can not exceed a certain critical value J_C:

$$J = J_C \sin(\varphi_1 - \varphi_2) \qquad (5.1)$$

The non-stationary Josephson effect appears at a nonzero voltage U_S on the contact. In this case a high-frequency alternating current percolates through the contact which frequency ν is proportional to the voltage:

$$\nu = \frac{2eU_S}{h} \qquad (5.2)$$

To understand a practical situation, we will consider the chain represented in fig. 1.14. When a constant superconducting current flows through the contact (stationary effect of Josephson) the voltage on contact is equal to zero, i.e. all U_S falls on the resistance R. This regime can exist if the current (equal to U_S/R) does not exceed the critical value I_C. Thus, the stationary effect of Josephson takes place if $U_S < I_C R$. If $U_S > I_C R$, the generation of high-frequency current begins. Then the mathematical description of a chain becomes very difficult.

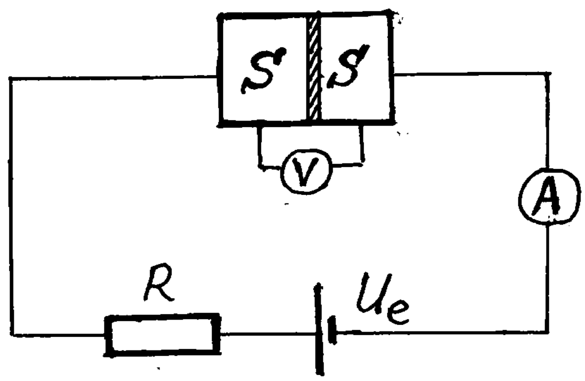

Fig. 1.14. The scheme for demonstration of Josephson effects.

From the point of view of quantum mechanics, all Cooper pairs are in the same state. This macroscopic filling of one state is the cause of the Josephson effects. Because all the pairs are in the same state, they must be consistent for all parameters, in particular, the phases. This strong correlation by phase applies to very large (virtually unlimited) distance.

Josephson equation (5.1) and (5.2) follow from the basic equations for a weakly coupled quantum systems. Let the systems are described by wave functions ψ_1 and ψ_2. If the systems are completely isolated, the change in the wave functions is described by equations

$$\frac{\partial \psi_1}{\partial t} = -\frac{i}{\hbar} E_1 \psi_1 \qquad \frac{\partial \psi_2}{\partial t} = -\frac{i}{\hbar} E_2 \psi_2 \tag{5.3}$$

If the systems are weakly linked, the time dependence of ψ_1 effects on ψ_2 and vice versa. This effect is taken into account by the following equations

$$\frac{\partial \psi_1}{\partial t} = -\frac{i}{\hbar}\left(E_1 \psi_1 + K \psi_2\right) \tag{5.4}$$

$$\frac{\partial \psi_2}{\partial t} = -\frac{i}{\hbar}\left(E_2 \psi_2 + K \psi_1\right) \tag{5.5}$$

The existence of connection means the ability to exchange Cooper pairs between the superconductors 1 and 2. The intensity of the exchange is determined by the constant *K*.

Functions ψ_1 and ψ_2 describe the states with macroscopic filling. Then the square of the amplitude can be regarded as the concentration of the Cooper pairs. In this case, we can write

$$\psi_1 = \sqrt{n_{C1}} \cdot e^{i\varphi_1}; \qquad \psi_2 = \sqrt{n_{C2}} \cdot e^{i\varphi_2} \tag{5.6}$$

Substituting these wave functions in (5.4) and (5.5), we obtain

$$\frac{\dot{n}_{C1}}{2\sqrt{n_{C1}}} e^{i\varphi_1} + i\sqrt{n_{C1}} e^{i\varphi_1} \dot{\varphi}_1 = -\frac{i}{\hbar}\left(E_1 \sqrt{n_{C1}} e^{i\varphi_1} + K\sqrt{n_{C2}} e^{i\varphi_2}\right) \tag{5.7}$$

$$\frac{\dot{n}_{C2}}{2\sqrt{n_{C2}}} e^{i\varphi_2} + i\sqrt{n_{C2}} e^{i\varphi_2} \dot{\varphi}_2 = -\frac{i}{\hbar}\left(E_2 \sqrt{n_{C2}} e^{i\varphi_2} + K\sqrt{n_{C1}} e^{i\varphi_1}\right) \tag{5.8}$$

Splitting into real and imaginary parts gives

$$\frac{\dot{n}_{C1}}{2\sqrt{n_{C1}}} = \frac{K}{\hbar}\sqrt{n_{C2}} \sin(\varphi_2 - \varphi_1) \; ; \tag{5.9}$$

$$\frac{\dot{n}_{C2}}{2\sqrt{n_{C2}}} = \frac{K}{\hbar}\sqrt{n_{C1}} \sin(\varphi_1 - \varphi_2) \tag{5.9 '}$$

$$\sqrt{n_{C1}}\dot{\varphi}_1 = -\frac{1}{\hbar}\left(E_1\sqrt{n_{C1}} + K\sqrt{n_{C2}}\cos(\varphi_2 - \varphi_1)\right) \tag{5.10}$$

$$\sqrt{n_{C2}}\dot{\varphi}_2 = -\frac{1}{\hbar}\left(E_2\sqrt{n_{C2}} + K\sqrt{n_{C1}}\cos(\varphi_1 - \varphi_2)\right) \tag{5.10'}$$

At an exchange of Cooper pairs between the systems 1 and 2 the condition $\dot{n}_{C1} = -\dot{n}_{C2}$ takes place. When superconductors are the same, $n_{C1} = n_{C2}$. Then from (5.9) and (5.9 '), we obtain

$$\dot{n}_{C1} = \frac{2K}{\hbar} n_{C1} \sin(\varphi_2 - \varphi_1) = -\dot{n}_{C2} \tag{5.11}$$

The time variation of particle concentration in the superconductor 1, multiplied by its volume V, gives the change of the number of particles, i.e. particle flow through the junction. An electric current is obtained by multiplying the flow of particles by the charge of the particle, i.e. 2e. Then we get the Josephson equation (5.1)

$$J = J_C \sin(\varphi_1 - \varphi_2),$$

where $J_C = \frac{4Ke}{\hbar} V n_C$.

From (5.10) and (5.10'), we obtain the differential equation

$$\frac{d}{dt}(\varphi_2 - \varphi_1) = \frac{1}{\hbar}(E_1 - E_2) \quad (5.12)$$

When $E_1 = E_2$ the phase difference is constant over time. If a voltage U is applied between superconductors, then $E_1 - E_2 = 2eU$ and the phase difference increases linearly with time

$$\varphi_2 - \varphi_1 = \frac{2eU}{\hbar} t + \varphi_0 \quad (5.13)$$

This means that alternating current flows through the contact

$$J = J_C \sin(\frac{2eU}{\hbar} t + \varphi_0) \quad (5.14)$$

the frequency of which is equal to $\nu = \frac{2eU}{h}$. When the voltage U on the contact equals 1 mV we have $\nu = 4{,}85 \cdot 10^{11}$ Hz.

Josephson junctions are also called “weak links”, and events associated with them are called "weak superconductivity." The weak link can be obtained also by a decrease in the contact area, for example, by pressing the sharp tip of a superconductor to a superconducting surface.