§5.2. Interference of stationary superconducting currents.

Let us analyze some experimental results of stationary Josephson effect.

5.2.1. Superconducting interferometer

We consider a closed contour Γ passing inside a superconducting ring containing two tunnel contacts and crossing them in points 1 and 2 (fig. 5.2)

A phase change at round of a contour is defined by a relation

$$\Delta\varphi = \oint_{\Gamma} \vec{\nabla}\varphi \cdot d\vec{l} + \varphi_1 - \varphi_2 \qquad (5.15)$$

where φ_1 and φ_2 - jumps of a phase on contacts T_1 and T_2, i.e. differences of values of a phase on the different sides of a contact. The current flowing through the interferometer is equal

$$J = J_{C1}\sin\varphi_1 + J_{C2}\sin\varphi_2 \qquad (5.16)$$

One of Ginzburg-Landau equations (3.16) has a form

$$\vec{j} = \frac{e}{m} n_S (\hbar\vec{\nabla}\varphi - 2e\vec{A}) \qquad (5.17)$$

where e and m - the charge and the mass of an electron, n_S - concentration of superconducting electrons. Finding from (5.17) $\vec{\nabla}\varphi$ and taking the integration contour in the superconductor depth where $j = 0$, we obtain

$$\oint_{\Gamma} \vec{\nabla}\varphi \cdot d\vec{l} = \frac{2e}{\hbar}\oint_{\Gamma} \vec{A} \cdot d\vec{l} = \frac{2e}{\hbar}\int \vec{B} \cdot d\vec{S} = 2\pi\frac{\Phi}{\Phi_0} \qquad (5.18)$$

where Φ is the magnetic flux penetrating the section of a ring, $\Phi_0 = \dfrac{\pi\hbar}{e}$ - the magnetic flux quantum.

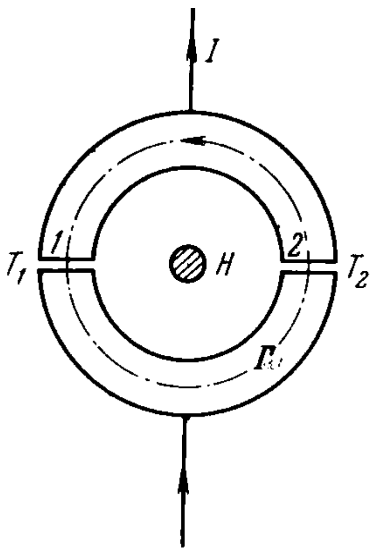

Fig. 5.2. Scheme of the superconducting interferometer

At round of a ring the wave functions have to be unambiguous, i.e. change of a phase has to be multiple 2π : $\Delta\varphi = 2\pi k$. From (5.15) we obtain a condition of quantization of a fluxoid

$$\underbrace{\varphi_1 - \varphi_2 + 2\pi \frac{\Phi}{\Phi_0}}_{Fluxoid} = 2\pi k \tag{5.19}$$

Having entered a new variable $\psi = \varphi_1 + \frac{\pi\Phi}{\Phi_0}$, we get from (5.16)

$$J = J_{C1}\sin(\psi - \frac{\pi\Phi}{\Phi_0}) + J_{C2}\sin(\psi + \frac{\pi\Phi}{\Phi_0}) = J_m \sin(\psi - \alpha) \tag{5.20}$$

where
$$J_m = \sqrt{(J_{C1} - J_{C2})^2 + 4J_{C1}J_{C2}\cos^2(\pi\Phi/\Phi_0)}, \tag{5.21}$$

$$\alpha = arctg\left(\frac{J_{C1} - J_{C2}}{J_{C1} + J_{C2}} tg\frac{\pi\Phi}{\Phi_0}\right) \tag{5.22}$$

From (5.20) we can see that the greatest possible current via the interferometer is equal to J_m. Changing at the fixed value of a magnetic field the value of the current J through the interferometer, for example, by means of the scheme of fig. 5.1, and finding the value J at which there is a transition to non-stationary effect of Josephson, we can find J_m. Changing a magnetic field, it is possible to draw the dependence of J_m on a magnetic flux through the opening of the interferometer. As it is clear from (5.21), it has to be periodic function with the period equal to Φ_0. Since Φ_0 is very small, such device can be used for registration of very small magnetic fields.

If the interferometer consists of identical contacts ($J_{C1} = J_{C2} = J_C$), the expression for J_m takes the form

$$J_m = 2J_C\left|\cos\frac{\pi\Phi}{\Phi_0}\right| \tag{5.23}$$

from where it is clear that the critical current of the interferometer J_m becomes zero every time when the flux Φ is equal to half-integer number of flux quantum Φ_0 (fig. 5.3a). If $J_{C1} \neq J_{C2}$, J_m doesn't equal zero at any values of a flux Φ and oscillates between the minimum and maximum values $|J_{C1} - J_{C2}|$ and $J_{C1} + J_{C2}$ (fig. 5.3b).

We will note important circumstance. Nature of behavior of current doesn't depend on how the field in a ring is distributed. Only a value of a magnetic flux Φ is important. In particular, the magnetic field can be concentrated entirely within some area, smaller than the ring openings, and equal to zero in the location a superconductor.

We can realize such situation, for example, having installed into the interferometer the long solenoid outside of which the field is absent. In this case the impact on current is carried out entirely by vector potential $\vec{A}$. Thus, in quantum mechanics the vector potential plays especially essential role and observed characteristics are defined not only by the magnetic field induction $\vec{B}$ or $\vec{H}$, but also by the vector potential $\vec{A}$. This is a purely quantum effect and it is difficult to offer an evident explanation for it within classical physics.

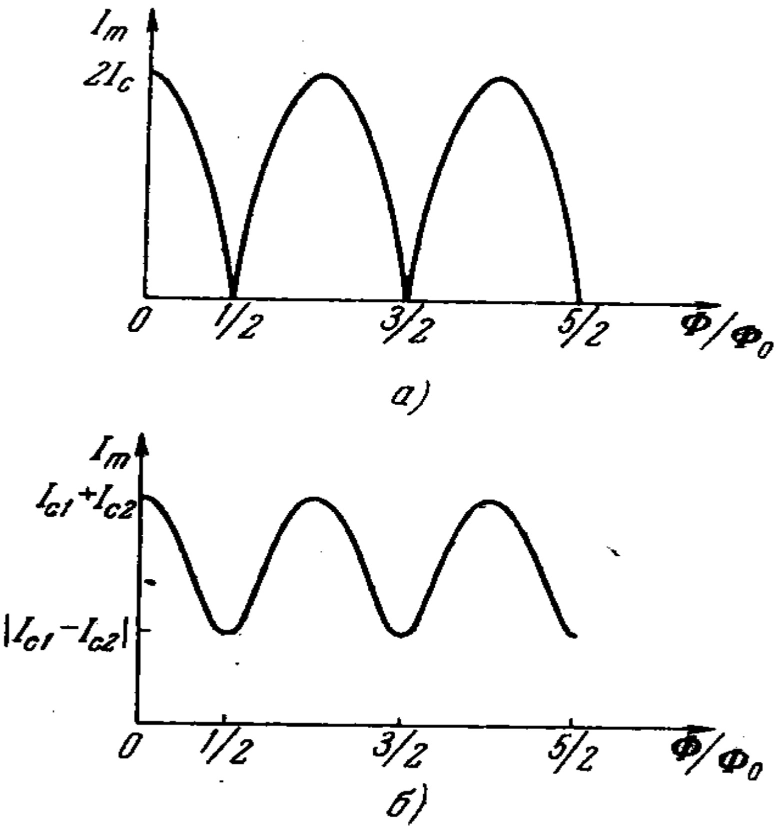

Fig. 5.3. Dependence of critical current of the interferometer on a magnetic field

5.2.2. A superconducting ring with a weak link

We consider features of behavior of a single tunnel junction in the closed superconducting chain (fig. 5.4). If the critical current of the contact is rather high, the magnetic field in a ring opening can't be considered equal to the external.

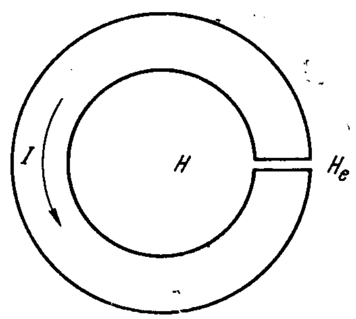

Fig. 5.4. A superconducting ring with a weak link.

The equation of a quantum interference (5.19) takes the form

$$2\pi \frac{\Phi}{\Phi_0} + \varphi = 2\pi k \qquad (5.24)$$

where φ is a difference of phases on a barrier, k - an integer.

The current through the contact is connected with a phase jump on it

$$J = J_C \sin\varphi \qquad (5.25)$$

The expression for the magnetic flux Φ through the ring is added to these two equations

$$\Phi = \Phi_e + LJ, \qquad (5.26)$$

where $\Phi_e = B_e S$ is a flux of an external field B_e through the ring, L - the inductance of the ring, S - its area. The condition (5.26) corresponds to the fact that the field in the opening of the ring consists of the external and created by current J flowing in the ring.
From the equations (5.24) - (5.26) we obtain

$$\Phi + LJ_C \sin\frac{2\pi\Phi}{\Phi_0} = \Phi_e \qquad (5.27)$$

The dependences of Φ and J on Φ_e are given in fig. 5.5. It is convenient to build the graph $\Phi_e(\Phi)$, and then to turn it on 90 degrees.

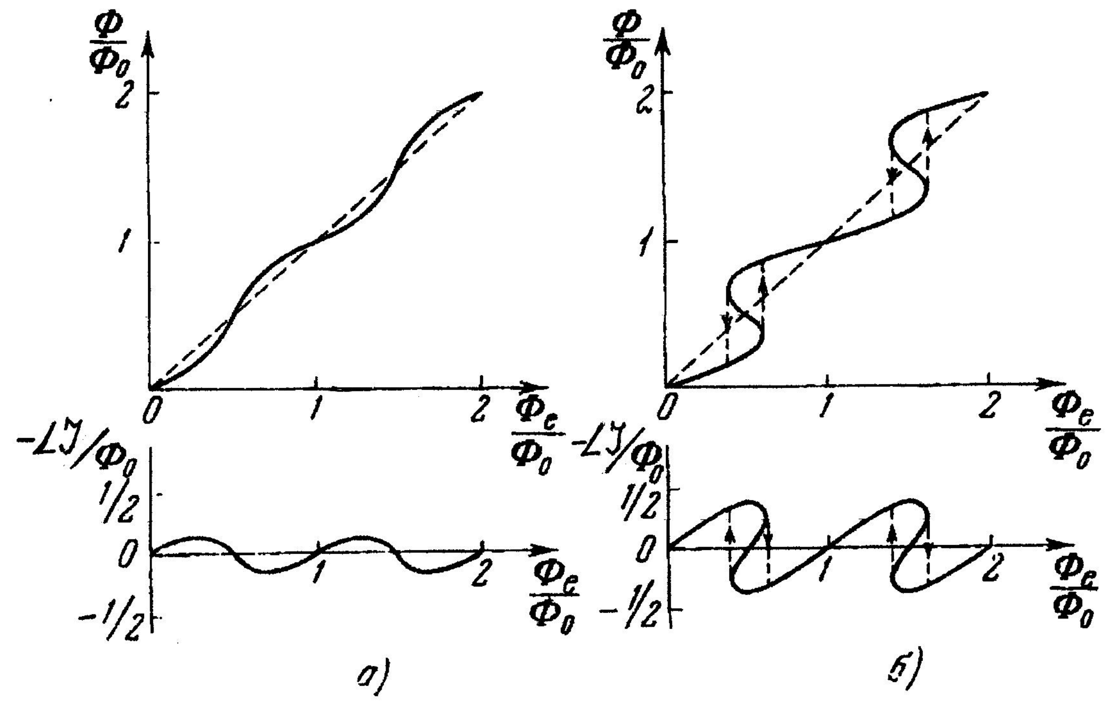

Fig. 5.5. Dependence of the flux Φ through the ring and the circulating current J on an external flux Φ_e: a) $J_C < J_C^{кр}$, b) $J_C > J_C^{кр}$.

At small values of J_C the dependence $\Phi(\Phi_e)$ does not differ much from a straight line, and the dependence $J(\Phi_e)$ - from a sinusoid. When J_C increases the character of these curves changes. For example, if J_C exceeds a certain critical value $J_C^{кр}$, these dependences become ambiguous so that to one value of Φ_e there can correspond several values of Φ and J (fig. 5.5b). Critical value $J_C^{кр}$ can be found from a condition $\partial\Phi/\partial\Phi_e = \infty$:

$$J_C^{кр} = \frac{\Phi_0}{2\pi L} \qquad (5.28)$$

At $J_C > J_C^{кр}$ the dependences $\Phi(\Phi_e)$ and $J(\Phi_e)$ have jumps at some values of Φ_e. At increase and decrease of the external flux Φ_e these dependences have various forms, i.e. there is a hysteresis. The direction of transition at a hysteresis is shown by arrows in fig. 5.5b.

The similar effect has to exist in superconducting interferometers with two tunnel junctions if the value of critical current J_C is comparable with the parameter Φ_0 / L where L is the inductance of a loop of the interferometer. The consideration of operation of such interferometers, carried out in the previous paragraph, concerned the case $J_C << \Phi_0 / L$.

§5.3. Interaction of alternating Josephson current with the external electromagnetic radiation - Shapiro's steps.

Discovery of non-stationary effect of Josephson gave the chance of creation of a new type of generators of electromagnetic radiation with very high frequency of radiation which value is regulated by the operating voltage.

Historically, however, some indirect experimental confirmations of the existence of this effect were originally obtained. In these experiments the features on volt-ampere characteristics arising owing to interaction of alternating superconducting current with an external electromagnetic microwave radiation were studied.

In the experiments of Shapiro with employees (1963), a tunnel contact of Al-Sn was located in the microwave resonator in which microwave fluctuations of frequency ν were created. The constant voltage U_0 was applied to the contact. Under the influence of a microwave field on a volt-ampere characteristics some horizontal sites - steps - were observed. Their positions correspond to a relation $2eU_0 = nh\nu$, where n are arbitrary integers (fig. 5.6).

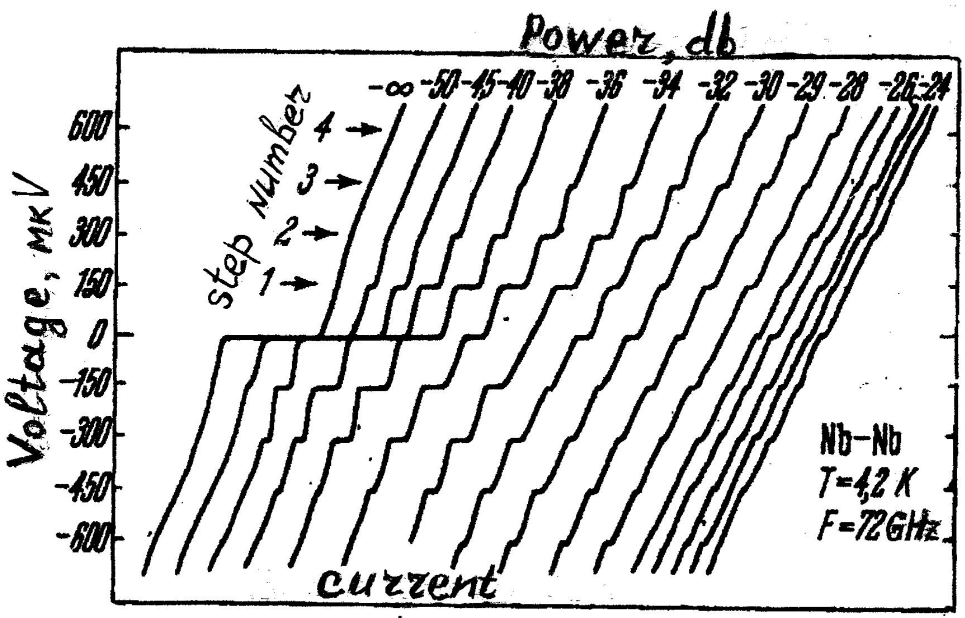

Fig. 5.6. Volt-ampere characteristics of Josephson contact of Nb-Nb in the presence of the microwave radiation of various power with a frequency of 72 GHz.

For clarification of the reason of this effect we will consider simply that the microwave field causes the appearance on the contact of the additional voltage oscillating with a frequency ν what leads to the modulation of frequency of Josephson current ν_J in accordance with (5.2)

$$\nu_J = \frac{2e}{h}\left[U_0 + u\cos(2\pi\nu t + \theta)\right] \qquad (5.29)$$

where u is an amplitude of an oscillating additive proportional to the intensity of a microwave field.

The total current through the contact consists of a Josephson current and a current of not coupled electrons, equal to U/R, where $U = U_0 + u\cos(2\pi\nu t + \theta)$ is a full voltage on the contact, R – the contact resistance. Since the voltage on the contact depends on time, instead of (5.13) we will come to $\varphi_2 - \varphi_1 = \int \frac{2eU}{\hbar} dt + \varphi_0$.

Thus, the current is equal to

$$J = J_C \sin\left(\int_0^t 2\pi\nu_J\, dt + \varphi_0\right) + \frac{U}{R} = J_C \sin\left[\frac{2e}{\hbar} U_0 t + \frac{2eu}{h\nu}\sin(2\pi\nu t + \theta) + \varphi_0\right] + \frac{U}{R} \quad (5.30)$$

where φ_0 - the initial value of a phase jump on the contact.

After a number of transformations with use of series

$$\sin(z\sin x) = 2\sum_{k=0}^{\infty} J_{2k+1}(z)\sin(2k+1)x$$

$$\cos(z\sin x) = J_0(z) + 2\sum_{k=1}^{\infty} J_{2k}(z)\cos 2kx$$

the expression (5.30) takes a form

$$J = J_C\left\{J_0\left(\frac{2eU}{h\nu}\right)\sin\left(\frac{2e}{\hbar}U_0 t + \varphi_0\right) + \sum_{n=1}^{\infty} J_n\left(\frac{2eu}{h\nu}\right)\left(\sin\left[\frac{2e}{\hbar}U_0 t + \varphi_0 + n(2\pi\nu t + \theta)\right]\right.\right.$$
$$\left.\left. + (-1)^n \sin\left[\frac{2e}{\hbar}U_0 t + \varphi_0 - n(2\pi\nu t + \theta)\right]\right)\right\} + \frac{U_0 + u\cos(2\pi\nu t + \theta)}{R} \quad (5.31)$$

where $J_n(x)$ - the Bessel function of n-th order.

From (5.31) we can see that the total current oscillates in time with very high frequency ν or with a Josephson frequency $2eU_0/h$. The measured value of the current is an average on time from the expression (5.31). It is easy to see that this value is equal to $\bar{J} = \frac{U_0}{R}$ at all values of U_0, except those at which $2eU_0/\hbar = 2\pi n\nu$, i.e. when the Josephson frequency is multiple of the microwave field frequency. At these values of U_0 one of the members of the sum stops being oscillating and makes a constant contribution to the value of the current. Thus the current becomes equal to

$$\bar{J} = \frac{U_0}{R} + (-1)^n J_n\left(\frac{2eu}{\hbar\nu}\right)\sin(\varphi_0 - n\theta). \quad (5.32)$$

Since $\varphi_0 - n\theta$ can assume arbitrary values, at the values of U_0, corresponding to the condition $2eU_0 = nh\nu$, the current can take values of a certain range. It means that at these voltages on the current-voltage characteristic there exist horizontal steps, what is confirmed by the experimental curves of Figure 5.6. This figure shows clearly not only the coincidence of the steps with the theoretical predictions, but satisfying quantitative agreement between the periodic change of the length of the steps with increasing microwave power and the dependence of the Bessel function on u.